MATEMATIKA FUZZY DAN APLIKASINYA DALAM KEHIDUPAN NYATA

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What is Fuzzy Mathematics?

- Fuzzy mathematics forms a branch of mathematics related to fuzzy set theory and fuzzy logic.
  - Fuzzy sets (uncertain sets) are somewhat like sets whose elements have degrees of membership (Lotfi A. Zadeh, 1965).
  - Fuzzy logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1 both inclusive.
- Fuzziness can be found in many areas of daily life, such as in engineering, medicine, meteorology, manufacturing, economy, and others. It is particularly frequent, however, in all areas in which human judgment, evaluation, and decisions are important.

Some fields of mathematics using fuzzy set theory

- Fuzzy subgroupoids and fuzzy subgroups were introduced in 1971 by A. Rosenfeld .
- Fuzzy fields and fuzzy Galois theory are published in a 1998 paper. (Mordeson & Malik)
- Fuzzy topology was introduced by C.L. Chang in 1968
- Fuzzy geometry were introduced by Tim Poston in 1971, A. Rosenfeld in 1974, by J.J. Buckley and E. Eslami in 1997 and by D. Ghosh and D. Chakraborty in 2012-14.
- Basic types of fuzzy relations were introduced by Zadeh in 1971.
- Fuzzy graphs have been studied by A. Kaufman, A. Rosenfel, and by R.T. Yeh and S.Y. Bang.

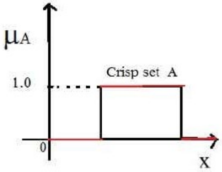
Fuzzy sets

- In mathematics fuzzy sets have triggered new research topics in connection with category theory, topology, algebra, analysis.
- Fuzzy sets are also part of a recent trend in the study of generalized measures and integrals, and are combined with statistical methods.
- In decision and organization sciences, fuzzy sets has had a great impact in preference modeling and multicriteria evaluation, and has helped bringing optimization techniques closer to the users needs.
- Applications can be found in many areas such as management, production research, and finance. Moreover concepts and methods of fuzzy set theory have attracted scientists in many other disciplines pertaining to human-oriented studies such as cognitive psychology and some aspects of social sciences.

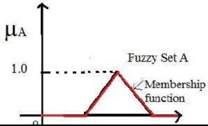
- The concept of a set and set theory are powerful concepts in mathematics. However, the principal notion underlying set theory, that an element can (exclusively) either belong to set or not belong to a set, makes it well nigh impossible to represent much of human discourse.
- How is one to represent notions like:
  large profit
  high pressure
  tall man
  moderate temperature

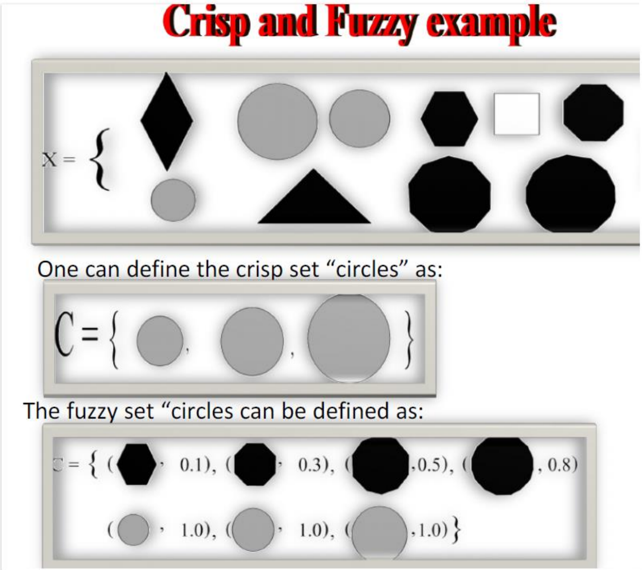
Crisp Set and Fuzzy Sets

- A crisp set, A, can be defined as a set which consists of elements with either full or no membership at all in the set. Each item in its universe is eitherin the set, or not.

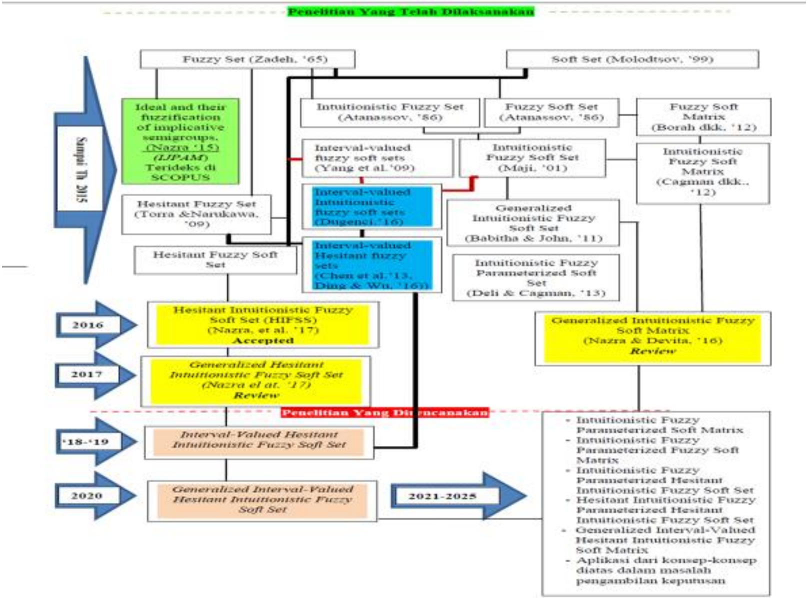


- A fuzzy set is defined as a class of objects with acontinuum of grades of membership. It is characterized by a membership function or characteristic function that assigns to each member of the fuzzy set a degree of membership inthe unit interval [0,1].




Crisp and Fuzzy example
X = {
One can define the crisp set “circles” as:
C = {
}
The fuzzy set “circles can be defined as:
= { (
0.1), (
0.3), (
,0.5), (
, 0.8)
(
1.0), (
1.0), (
,1.0) }

Topics in Fuzzy Sets



Soft Set

Definition (Mololodtsov, 1999)

Let U be an initial universe set and let E be a set of parameters. A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set P(U), i.e. $F\colon E \to P(U)$, where P(U) is the power set of U.

Example

Let U $=$ {h1, h2, h3, h4, h5, h6} be a set of houses under consideration where E $=$ {e1, e2, e3, e4, e5} and A $=${e1, e2, e3, e4} be a subset of parameters for selection of the house. Let e1 stands for expensive houses, e2 stands for wooden houses. e3 stands for houses located in green surroundings, e4 stands for houses located in the urban area, e5 stands for the low cost houses. Let (F, A) be the soft set to categorize the houses with respect to parameters given by set A, such that
F(e1) $=${ h1, h3}, F(e2) $=$ {h1, h3, h6}, F(e3) $=${h1, h3, h4, h5}, F(e4) $=${h1, h2, h3}

Soft Set-example

For computer applications it is more appropriate to represent a soft set in tabular form

	h1	h2	h3	h4	h5	h6
e1	1	0	1	0	0	0
e2	1	0	1	0	0	1
e3	1	0	1	1	1	0
e4	1	1	1	0	0	0

AN APPLICATION OF SOFT SET THEORY (Maji, 2002)

- Let U = {h1, h2, h3, h4, h5, h6}, be a set of six houses,
- E = {expensive=e1; beautiful=e2; wooden=e3; cheap=e4; in the green surroundings=e5: modern=e6; in good repair=e7: in bad repair=e8}, be a set of parameters.
- Consider the soft set (F, E) which describes the attractiveness of the houses, given by (F, E) = {expensive houses = ∅, beautiful houses = {h1, h2, h3, h4, h5, h6}, wooden houses = {h1, h2, h6}, modern houses = { h1 , h2, h6}, in bad repair houses = { h2, h4, h5}, cheap houses = { h1, h2, h3, h4, h5, h6}, in good repair houses ={ h1, h3, h6}, in the green surroundings houses = {h1, h2, h3, h4, h6} }.

- Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters beautiful, wooden, cheap, in the green surroundings, in good repair,

	beautiful	wooden	cheap	in good repair	in the green surroundings
h1	1	0	1	1	1
h2	1	0	1	0	1
h3	1	0	1	1	1
h4	1	1	1	0	1
h5	1	1	1	0	0
h6	1	1	1	1	1

- The problem is to select the house which is most suitable with the choice parameters of Mr. X.

1^{st} **Method: Choice Value of an Object** h_i

1. input the soft set (F, E),
2. input the set P of choice parameters of Mr. X which is a subset of E,
3. choose a soft subset (F, P) of (F, E).
4. Count the choice value c_i of an object $h_i \in U$, given by $c_i = \sum_j h_{ij}$, where h_{ij} are the entries in the table.
5. find k, for which $c_k = max\ c_i$. Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X by using his option.

An Application of SS-cont

	beautiful	wooden	cheap	in good repair	in the green surroundings	Choose Value
h1	1	0	1	1	1	4
h2	1	0	1	0	1	3
h3	1	0	1	1	1	4
h4	1	1	1	0	1	4
h5	1	1	1	0	0	3
h6	1	1	1	1	1	5

Decision: Mr. X can buy the house h6

2^{nd} **Method: Weighted Choice Value of an Object** h_i

1. input the soft set (F, E),
2. input the set P of choice parameters of Mr. X which is a subset of E,
3. choose a soft subset (F, P) of (F, E).
4. Count the weighted choice value c_i of an object $h_i \in U$, given by $c_i = \sum_j d_{ij}$, where $d_{ij} = w_j \times h_{ij}$, w_j is the j^{th} weight decided by Mr. X,.
5. find k, for which $c_k = max\ c_i$. Then h_k is the optimal choice object. If k has more than one value, then any one of them could be chosen by Mr. X by using his option.

An Application of SS-cont

U	$e_1, w_1 = .8$	$e_2, w_2 = .3$	$e_4, w_4 = .9$	$e_5, w_5 = .8$	Choice Value
h_1	1	1	1	1	$c_1 = 2.8$
h_2	1	1	1	0	$c_2 = 2.0$
h_3	1	0	1	1	$c_3 = 2.5$
h_4	1	0	1	0	$c_4 = 1.7$
h_5	1	0	0	0	$c_5 = 0.8$
h_6	1	1	1	1	$c_6 = 2.8$

N-SOFT SETS (Fatimah et. al, 2018)

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https://doi.org/10.1007/s00500-017-2838-6

CrossMark

FOUNDATIONS

N-soft sets and their decision making algorithms

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José Carlos R. Alcantud[4]

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Abstract In this paper, we motivate and introduce the concept of N-soft set as an extended soft set model. Some useful algebraic definitions and properties are given. We cite real examples that prove that N-soft sets are a cogent model for binary and non-binary evaluations in numerous kinds of decision making problems. Finally, we propose decision making procedures for N-soft sets.

Keywords N-soft set · Non-binary evaluation · Decision making · Choice value · Intersection and union

imprecision, or subjectivity. This paper expands the range of applications of one of the theories that can be used to deal with these characteristics, namely soft set theory. It was introduced by Molodtsov (1999), who also showed its applicability to various fields. Soft set does not require parameters specification. Instead, it accommodates all types of parameters as its benchmark. The parameters of a soft set can be numbers, words, sentences, functions, and so on. Thus, the soft set definition associates the pertinent attributes with information or knowledge about the elements in the universe.

N-SOFT SETS

LAS MEJORES

★★★★★ OBRA MAESTRA ★★ INTERESANTE
★★★★ MUY BUENA ★ REGULAR
★★★ BUENA ● MALA

		EL MUNDO	EL PAÍS	GUIA DEL OCIO	DECINE21.COM	ABC	CAHIERS	CINEMANIA	FOTOGRAMAS	CINEYTEATRO.ES
		F.MARINERO A. BERMEJO A. LUCHINI LUIS MARTÍNEZ	JAVIER OCAÑA	ROBERTO PIORNO	JOSÉ MARÍA ARESTÉ	J.CORTIJO O.RODRÍGUEZ MARCHANTE	CARLOS F. HEREDERO	CARLOS MARAÑÓN	TONI ULLED	MARÍA JOSÉ SÁNCHEZ LERCHUNDI
1	SING STREET	★★★★★			★★★★					★★★★
2	TARDE PARA LA IRA	★★★★	★★★★	★★★	★★★	★★★★	★★★★★	★★★★★	★★★★★	★★★★
3	KUBO Y LAS DOS CUERDAS MÁGICAS	★★★★		★★★★★	★★★	★★★★	★★★★			★★★★
4	ELLE	★★★★★			★★★		★★★★			★★★
5	EL HÓMBRE DE LAS MIL CARAS	★★★★	★★★★		★★★	★★★	★★★	★★★★	★★★★	★★★★
6	EL HOGAR DE MISS PEREGRINE	★★★★	★★		★★★★					★★★★
7	CAFÉ SOCIETY	★★★★	★★★★	★★	★★★	★★★	★★★★	★★★★		★★★★
8	NERUDA	★★★★★	★★★	★★★	★★	★★★	★★★★		★★★★	★★★
9	SPARROWS	★★★		★★	★★★★	★★★		★★★★	★★★★	★★★
10	EL PORVENIR	★★		★★★★		★★★★	★★★			
11	FLORENCE FOSTER JENKINS	★★★		★★★★	★★★	★★★				★★★
12	ESCUADRÓN SUICIDA	★★		★★★		★★★		★★★	★★★★	★★★★
13	CAPTAIN FANTASTIC	★				★★★★	★★★			★★★★
14	NO RESPIRES	★★★	★★★	★★★		★★★				★★★
15	MASCOTAS	★★★	★★	★★				★★★		★★★★★
16	CIGÜEÑAS	★★								★★★★
17	JUEGO DE ARMAS	★★★		★★	★★★	★★★				★★★★
18	SUBURRA	★★★		★	★★	★★★		★★★★	★★★	★★★★
19	LA RECONQUISTA	★★★		★★★	★★★		★★★			★★
20	EL TIEMPO DE LOS MONSTRUOS	★★★★		★★			★★			★★★
21	LA VACA	★★						★★★		★★★
22	LOS SIETE MAGNÍFICOS	★★			★★★	★★		★★★		★★★
23	LOS HOMBRES LIBRES DE JONES	★	★★★	★★★		★★★		★★★	★★★	★★
24	LA ESTACIÓN DE LAS MUJERES	★★★		●	★★	★★★	★★★	★★★★	★★	★★
25	BRIDGET JONES' BABY	★				★★★				★★★

CONFECCIONADA MEDIANTE LOS JUICIOS DE LOS CRÍTICOS QUE FIGURAN EN EL ENCABEZAMIENTO. EL ORDEN DE LAS PELÍCULAS SE ESTABLECE SEGÚN LA MEDIA ARITMÉTICA DE LAS ESTRELLAS RECIBIDAS Y DEL NÚMERO DE CRÍTICOS QUE LAS VALORAN

Example 5 We draw a reduced 'cinema table' from Fig. 1 and then transfer its information to the language of 6-soft set. Let U be the universe of movies, U ={u_1 =Tarde para la ira, u_2 =Kubo y las dos cuerdas magicas, u_3 =Café society, u_4 =Neruda, u_5 =La estación de las mujeres}. Let E be the set of attributes 'evaluations of movies by media,' and $A \subseteq E$ be such that A ={ a_1 =El mundo, a_2 =Guia del ocio, a_3 = Decine21.com, a_4 = ABC, a_5 = Cahiers, a_6 = Cineyteatro.es }. In relation to these elements, a 6-soft set can be defined from Table 2, where five stars means 'obra maestra' (masterpiece), four stars means 'muy buena' (very good), three stars means 'buena' (good), two stars means 'interesante' (interesting), one star means 'regular' (average), and big dot means 'mala' (bad).

Table 2 Information extracted from the real data in Fig. 1

U	a_1	a_2	a_3	a_4	a_5	a_6
u_1	****	***	***	****	*****	****
u_2	****	*****	***	****	****	****
u_3	****	**	***	***	****	****
u_4	*****	***	**	***	****	***
u_5	***	•	**	***	***	**

It can be expressed as a 6-soft set (cf., Table 3)

Table 3 The 6-soft set in Example 5

$(F, A, 6)$	a_1	a_2	a_3	a_4	a_5	a_6
u_1	4	3	3	4	5	4
u_2	4	5	3	4	4	4
u_3	4	2	3	3	4	4
u_4	5	3	2	3	4	3
u_5	3	0	2	3	3	2

Algorithm 2 *The algorithm of EWCVs.*

1. *Input* $U = \{u_1, \ldots, u_p\}$ *and* $A = \{a_1, \ldots, a_q\}$, *and a weight* w_j *for each parameter* j.
2. *Input the* N*-soft set* (F, A, N), *with* $R = \{0, 1, \ldots, N-1\}$, $N \in \{2, 3, \ldots\}$, *so that* $\forall\, u_i \in U, a_j \in A, \exists!\, r_{ij} \in R$.
3. *For each* u_i, *compute its EWCV* $\sigma_i^w = \sum_{j=1}^{q} w_j r_{ij}$.
4. *Find all indices* k *for which* $\sigma_k^w = \max_{i=1,\ldots,p} \sigma_i^w$.
5. *The solution is any* u_k *from Step 4.*

extended weight choice values (EWCVs),

$(F, A, 6)$	a_1 $w_1 = 0.1$	a_2 $w_2 = 0.3$	a_3 $w_3 = 0.3$	a_4 $w_4 = 0.1$	a_5 $w_5 = 0.1$	a_6 $w_6 = 0.1$	σ_i^w
u_1	0.4	0.9	0.9	0.4	0.5	0.4	3.5
u_2	0.4	1.5	0.9	0.4	0.4	0.4	*4*
u_3	0.4	0.6	0.9	0.3	0.4	0.4	3
u_4	0.5	0.9	0.6	0.3	0.4	0.3	3
u_5	0.3	0	0.6	0.3	0.3	0.2	1.7

FUZZY SETS

- In order to explain the concepts of fuzzy sets, the basic idea in classical set theory must be understood.
- In mathematics, the concept of classical set is very simple.
  A set is a collection of well-defined objects.
  hese objects cover almost anything that can either belong or do not belong to the set.

The classical set A in the universe U, $A \subset U$ is normally characterised by the function $\mu_A(x)$, which take value 1 or 0, indicating whether or not $x \in U$ is a member of A:

$$\mu_A(x) = \begin{cases} 1 & for \quad x \in A \\ 0 & for \quad x \in A \end{cases} \tag{1}$$

Hence, $\mu_A(x) \in \{0, 1\}$. The function $\mu_A(x)$ as equation (1) takes only the value 1 or 0.

FUZZY SETS

- Ragin (2000) had a very simple explanation about fuzzy sets.
- He iterated that the basic idea behind fuzzy sets is to permit the scaling of membership scores and this allows partial or fuzzy membership.
- A membership score of 1 indicates full membership in a set; a score close to 1 (e.g., 0.8 or 0.9) indicates strong but partial membership in a set; scores less than 0.5 but greater than 0 (e.g., 0.2 and 0.3) indicate that objects are more out than in a set, but still weak members of the set;
  a score of 0 indicates full non membership in the set.
- Thus, fuzzy sets combine qualitative and quantitative assessment.

FUZZY SETS

- The fuzzy set theory can represent the uncertainty or vagueness inherent in the definition of linguistic variables (Zadeh, 1975).
- Age is one of the examples of a linguistic variable whose values are words like very young, middle age, and very old.
- Fuzzy set theory is becoming an alternative way in explaining the fuzzy phenomena in the real world.

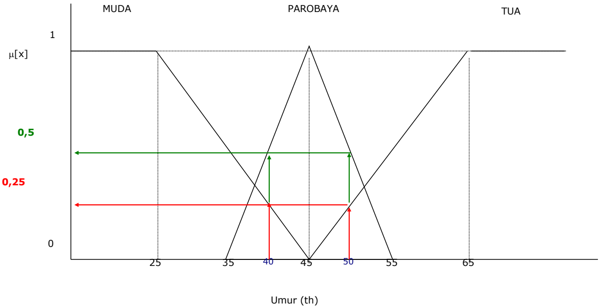
Assume that the function $\mu_A(x)$ may take values in the interval [0, 1]. In this way, the concept of membership is not any more crisp, but become fuzzy in the sense of representing partial belonging or degree of membership (Bojadziev, 1999). A fuzzy set R is defined by:

$$R = \{(x, \mu_R(x)) / x \in A, \mu_R(x) \in [0,1]\} \tag{2}$$

where $\mu_R(x)$ is a function called membership function; $\mu_R(x)$ specifies the grade or degree to which any element in A belongs to the fuzzy set R.

FS-example

Seseorang dapat masuk dalam 2 himpunan yang berbeda, MUDA dan PAROBAYA, PAROBAYA dan TUA, dsb. Seberapa besar eksistensinya dalam himpunan tersebut dapat dilihat pada nilai keanggotaannya.



Himpunan fuzzy untuk variabel Umur

FUZZY SETS and Its Operations

The **fuzzy set (FS)** μ **over a set** U is a set $\{(u, \mu(u)) | u \in U\}$ where $\mu \colon U \to [0, 1]$.
μ is called the membership function of X
$\mu(u)$ is called the membership value of u.

Definition

Given two FSs μ and ν over U.

- $\mu^c := \{(u, \mu(u)) | u \in U\}^c = \{(u, \mu^c(u)) | u \in U\}$, with $\mu^c(u) := 1 - \mu(u)$
- $\mu \cup \nu := \{(u, \mu(u) \star \nu(u)) | u \in U\}$, with $\mu(u) \star \nu(u) := max\{\mu(u), \nu(u)\}$
- $\mu \cap \nu := \{(u, \mu(u) \diamond \nu(u)) | u \in U\}$, with $\mu(u) \diamond \nu(u) := min\{\mu(u), \nu(u)\}$

Definition

Fuzzy set μ over U is called

- The null FS, denoted by $\emptyset := \{(u, 0) | u \in U\}$
- The universal FS, denoted by $\mathbf{1} := \{(u, 1) | u \in U\}$

Proposition

Given two FSs μ, ν and $w(u)$ over U.

- $(\mu^c)^c = \mu$
- $\mu \cup \nu = \nu \cup \mu$
- $\mu \cap \nu = \nu \cap \mu$
- $(\mu \cup \nu)^c = \nu^c \cap \mu^c$
- $(\mu \cap \nu)^c = \nu^c \cup \mu^c$
- $\mu \cap (\nu \cap w) = (\nu \cap \mu) \cap w$
- $\mu \cup (\nu \cup w) = (\nu \cup \mu) \cup w$

Definition (Maji, 2001)

U : a universal set, E : a paramater set. $A \subseteq E$.
A pair $(\mathbf{f_A}, \mathbf{A})$ is called **a fuzzy soft set over** U
if $f_A \colon A \to I^U$, $\quad I^U$: the collection of all fuzzy sets over U.

- Suppose that, Mr. X is interested to buy a house on the basis of his choice parameters beautiful, wooden, cheap, in the green surroundings, in good repair,

	beautiful	wooden	cheap	in good repair	in the green surroundings
h1	(1;0.3)	0	(1;0.3)	(1;0.3)	(1;0.3)
h2	(1;0.7)	0	(1;0.3)	0	(1;0.3)
h3	(1;0.4)	0	(1;0.3)	(1;0.3)	(1;0.3)
h4	(1;0.9)	(1;0.3)	(1;0.3)	0	(1;0.3)
h5	(1;0.1)	(1;0.3)	(1;0.3)	0	0
h6	(1;0.3)	(1;0.3)	(1;0.3)	(1;0.3)	(1;0.3)

- The problem is to select the house which is most suitable with the choice parameters of Mr. X.

(Nasef et. al, 2018)

- Suppose that Mr. X is interested to buy a house from among the set of houses U = {h1; h2; h3} on the basis of the set E = {e1 = cheap house, e2 =beautiful house, e3 =green surroundings, e4 = good location house} of selection criteria called the parameters.
- Suppose Mr. X is going to buy the house on his own preference weightage to the selection criteria.
- Now, to get the recent market information, that is, the performance evaluation matrix we construct the fuzzy soft sets, (F1,E), (F2,E), (F3,E) over the universe U.

$$(F_1, E) = \begin{array}{c c} & \begin{array}{ccc} h_1 & h_2 & h_3 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} & \begin{pmatrix} 0.7 & 0.6 & 0.3 \\ 0.5 & 0.4 & 0.7 \\ 0.7 & 0.5 & 0.5 \\ 0.4 & 0.7 & 0.8 \end{pmatrix} \end{array},$$

$$(F_2, E) = \begin{array}{c c} & \begin{array}{ccc} h_1 & h_2 & h_3 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} & \begin{pmatrix} 0.4 & 0.7 & 0.3 \\ 0.8 & 0.4 & 0.5 \\ 0.9 & 0.5 & 0.6 \\ 0.5 & 0.7 & 0.9 \end{pmatrix} \end{array},$$

$$(F_3, E) = \begin{array}{c c} & \begin{array}{ccc} h_1 & h_2 & h_3 \end{array} \\ \begin{array}{c} e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} & \begin{pmatrix} 0.2 & 0.9 & 0.4 \\ 0.5 & 0.7 & 0.4 \\ 0.4 & 0.7 & 0.8 \\ 0.6 & 0.4 & 0.8 \end{pmatrix} \end{array}.$$

Then by taking the average of the above three fuzzy soft sets, we get the performance evaluation matrix (or recent market survey information) as follow:

$$R = \begin{array}{c} \\ e_1 \\ e_2 \\ e_3 \\ e_4 \end{array} \begin{array}{c} \begin{array}{ccc} h_1 & h_2 & h_3 \end{array} \\ \begin{pmatrix} 0.433 & 0.733 & 0.333 \\ 0.600 & 0.500 & 0.533 \\ 0.667 & 0.567 & 0.633 \\ 0.500 & 0.600 & 0.833 \end{pmatrix} \end{array}.$$

Hence

$$R^T = \begin{array}{c} \\ h_1 \\ h_2 \\ h_3 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{pmatrix} 0.433 & 0.600 & 0.667 & 0.500 \\ 0.733 & 0.500 & 0.567 & 0.600 \\ 0.333 & 0.533 & 0.633 & 0.833 \end{pmatrix} \end{array}.$$

Next, suppose that the preference weightage of $Mr \cdot X$ to the different selection criteria is given by the following table:

$W =$

e_1	e_2	e_3	e_4
0.3	0.2	0.1	0.4

such that $\sum_{i=1}^{4} = W_j \leq 1$

Table 2

Thus, to get the comprehensive decision matrix D for $Mr \cdot X$, we multiply R^T by the preference weightage matrix and get matrix D as follows:

$$D = \begin{array}{c c} & \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{array}{c} h_1 \\ h_2 \\ h_3 \end{array} & \begin{pmatrix} 0.129 & 0.120 & 0.067 & 0.200 \\ 0.220 & 0.100 & 0.057 & 0.240 \\ 0.010 & 0.107 & 0.063 & 0.330 \end{pmatrix} \end{array}$$

The comparison table of the above comprehensive decision matrix is:

U	h_1	h_2	h_3
h_1	4	2	3
h_2	2	4	1
h_3	1	3	4

Table 3

Next, we compute the row-sum, column-sum from the comprehensive decision matrix and the score for each house h_i, $i = 1, 2, 3$ as follows:

	Row-sum	Column-sum	Score value
h_1	9	7	2
h_2	7	9	-2
h_3	8	8	0

Table 4: Scor table.

Obviously from the above table, we notice that the maximum score is 2, which is related to the house h_1. Therefore, the house h_1 is the best choice for $Mr \cdot X$.

Steps :	An algorithm
Step I :	Input the performance evaluation of houses by different counseling agencies as matrix.
Step II :	Find the average of the corresponding entries of all the matrices in Step (I).
Step III :	Multiply the weightage of different houses of the guardins to the corresponding entries of each row to get the comprehensive decision matrix.
Step IV :	Formulate the comparison table.
Step V :	Find the row-sums and column-sums of the comparison table.
Step VI :	Obtain the score for each product and the product with maximum score is recommended as the best choice.

Intuitionistic Fuzzy Soft Set/ IFSS

Definition (Atanassov, 1986)

Let U be a set. **An intuitionistic fuzzy set X over the set U** is defined as

$$X = \{(x, \mu_X(z), \gamma_X(z))|z \in U\}$$

where $\mu_X \colon U \to [0,1]$ and $\gamma_X \colon U \to [0,1]$, such that $0 \leq \mu_X(z) + \gamma_X(z) \leq 1$ for all $z \in U$.
$\mu_X(z)$ is called the membership value of z
$\gamma_X(z)$ is the non-membership value of z.

Definition (Maji et. al, 2004)

Let U be a set, E is the set of parameters, $A \subset E$. A pair (F,A) is called an **intuitionistic fuzzy soft set** over U, where F is a mapping given by $F \colon A \to \mathcal{IF}(\mathcal{U})$, $\mathcal{IF}(\mathcal{U})$ is a collection of all intuitionistic fuzzy sets over U.

Intuitionistic Fuzzy Soft Set-ex

(Yang & Qian, 2013)

- Let us consider an intuitionistic fuzzy soft set $\omega = (F, A)$ which describes the "attractiveness of houses" that Mr. X is considering for purchase.
- Suppose there are six houses in the domain U = {h1; h2; h3; h4; h5; h6} under consideration, and A ={ e1; e2; e3; e4; e5; e6} is a set of decision parameters.
  The ei (i = 1; 2; 3; 4; 5; 6) stand for the parameters : modern, cheap, beautiful, large, convenient traffc and in green surroundings, respectively.
- Table 1 gives the tabular representation of the intuitionistic fuzzy soft set $\omega = (F, A)$

Table 1

Intuitionistic fuzzy soft set $\varpi = (F, A)$

U	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
h_1	$(0.3, 0.5)$	$(0.6, 0.3)$	$(0.6, 0.3)$	$(0.6, 0.3)$	$(0.3, 0.5)$	$(0.5, 0.2)$
h_2	$(0.8, 0.0)$	$(0.8, 0.1)$	$(0.8, 0.0)$	$(0.6, 0.2)$	$(0.4, 0.1)$	$(0.6, 0.1)$
h_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.2, 0.6)$	$(0.2, 0.6)$	$(0.5, 0.3)$	$(0.2, 0.1)$
h_4	$(0.2, 0.7)$	$(0.2, 0.6)$	$(0.0, 0.9)$	$(0.0, 0.9)$	$(0.2, 0.4)$	$(0.1, 0.7)$
h_5	$(0.7, 0.0)$	$(0.8, 0.1)$	$(0.8, 0.0)$	$(0.7, 0.1)$	$(0.5, 0.3)$	$(0.7, 0.1)$
h_6	$(0.8, 0.0)$	$(0.8, 0.1)$	$(0.5, 0.4)$	$(0.7, 0.1)$	$(0.8, 0.1)$	$(0.5, 0.1)$

If we choose $\lambda = topbottom_{\varpi}$ where $topbottom_{\varpi} : A \to [0,1] \times [0,1]$ defined by $\mu_{topbottom_{\varpi}}(\varepsilon) = max\{\mu_{F(\varepsilon)}(x) | x \in U\}$ and $\nu_{topbottom_{\varpi}}(\varepsilon) = min\{\mu_{F(\varepsilon)}(x) | x \in U\}$

for all $\varepsilon \in A$, then $topbottom_{\varpi} = \{(\varepsilon_1, 0.8, 0), (\varepsilon_2, 0.8, 0.1), (\varepsilon_3, 0.8, 0), (\varepsilon_4, 0.7, 0.1), (\varepsilon_5, 0.8, 0.1), (\varepsilon_6, 0.7, 0.1)\}$.

By using Algorithm 1, we obtain the level soft set $L(\varpi; topbottom_{\varpi})$ with the choice values with tabular representation as in Table 2.

Table 1

Intuitionistic fuzzy soft set $\varpi = (F, A)$

U	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6
h_1	$(0.3, 0.5)$	$(0.6, 0.3)$	$(0.6, 0.3)$	$(0.6, 0.3)$	$(0.3, 0.5)$	$(0.5, 0.2)$
h_2	$(0.8, 0.0)$	$(0.8, 0.1)$	$(0.8, 0.0)$	$(0.6, 0.2)$	$(0.4, 0.1)$	$(0.6, 0.1)$
h_3	$(0.5, 0.4)$	$(0.5, 0.4)$	$(0.2, 0.6)$	$(0.2, 0.6)$	$(0.5, 0.3)$	$(0.2, 0.1)$
h_4	$(0.2, 0.7)$	$(0.2, 0.6)$	$(0.0, 0.9)$	$(0.0, 0.9)$	$(0.2, 0.4)$	$(0.1, 0.7)$
h_5	$(0.7, 0.0)$	$(0.8, 0.1)$	$(0.8, 0.0)$	$(0.7, 0.1)$	$(0.5, 0.3)$	$(0.7, 0.1)$
h_6	$(0.8, 0.0)$	$(0.8, 0.1)$	$(0.5, 0.4)$	$(0.7, 0.1)$	$(0.8, 0.1)$	$(0.5, 0.1)$

Table 2

Level soft set $L(\varpi; topbottom_{\varpi})$ with choice values

U	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	Choice value (c_i)
h_1	0	0	0	0	0	0	$c_1 = 0$
h_2	1	1	1	0	0	0	$c_2 = 3$
h_3	0	0	0	0	0	0	$c_3 = 0$
h_4	0	0	0	0	0	0	$c_4 = 0$
h_5	0	1	1	1	0	1	$c_5 = 4$
h_6	1	1	0	1	1	0	$c_6 = 4$

Algorithm 1.

(1) Input the (result) intuitionistic fuzzy soft set $\varpi = (F, A)$.

(2) Input a threshold intuitionistic fuzzy set $\lambda : A \to [0,1] \times [0,1]$ for decision making.

(3) Compute the level soft set $L(\varpi; \lambda)$.

(4) Present the level soft set $L(\varpi; \lambda)$ in tabular form and compute the choice value c_i of x_i, $\forall i$.

(5) The optimal decision is to select x_k if $c_k = max_i c_i$.

(6) If k has more than one value, then any one of x_k may be chosen.

Definition (Torra, 2010)

Let U be a set. **A Hesitant fuzzy set X over the set U** is defined as

$$X = \{(x, h(z)) | z \in U\}$$

where $h\colon U \to \mathcal{P}([0,1])$
$h(z)$ is called the membership value of z
$\mathcal{P}([0,1])$ is a collection of all subsets of [0,1].
$h(z)$ is called a hesitant fuzzy element of $z \in U$.

Definition (Babitha & John, 2013)

Let U be a set, E is the set of parameters, $A \subset E$. A pair (F,A) is called an **Hesitant fuzzy soft set** over U, where F is a mapping given by $F\colon A \to \mathcal{HF}(\mathcal{U})$, $\mathcal{HF}(\mathcal{U})$ is a collection of all Hesitant fuzzy sets over U.

Hesitant fuzzy soft set in decision making problems

Definition (Xia & Xu, 2011)

For a hesitant fuzzy element $h(x)$, $s(h) = \frac{1}{l(h)} \sum_{\gamma \in h(x)} \gamma$ is called the score function of $h(x)$ where $l(h)$ denotes number of values in $h(x)$.

Definition (Babitha & John, 2013)

Let (F, A) denotes hesitant fuzzy soft set, Then the fuzzy soft set (F_S, A) in which each entries in the fuzzy set $F_S(e)$ is the score function of the respective entries in the hesitant fuzzy set $F(e)$ is called as score matrix.

Definition (Babitha & John, 2013)

The table obtained by calculating the average of $F_S(e_i)$ for each u_j is called as decision table.

Algorithm

1. Input the hesitant fuzzy soft set (F, A)
2. Obtain the score matrix (F_S, A) corresponds to (F, A)
3. Calculate the average of $F_S(e_i)$ for each u_j and let it be denoted as a_j. This is the decision table
4. Select the optimal alternative u_k if $a_k = \max_j a_j$
5. If k has more than one value then any one of u_k may be chosen.

Remark 4.5 In decision making problems further representational capability can be added by associating with each parameter e_i a value $w_i \in [0, 1]$ called its weight. In the case of multi-criteria decision making, these weights can be used to represent the different importance of the concerned criteria. In this case there is a small change in the above algorithm. In step 3 instead of average we take weighted average

$$\frac{\sum_{i=1}^{n} F_S(e_i)\, w_i}{n}$$

and follows the next step.

Hesitant fuzzy soft set in decision making problems

Let us consider decision-making problem of allocating a particular job to the best possible person who fulfills the requirements of the job. Selection is done by the interview board consisting of three members. Let U = $\{u_1, u_2, u_3, u_4\}$ be crisp set of four persons for the job. Let A= {enterprising, confident, willing to take risks, hardworking} be the set of parameters which represents the criteria for the problem. Let A can be represented as A =$\{e_1, e_2, e_3, e_4\}$ The problem is the selection of best person who satisfy the criteria to utmost extent.

All the available information on these candidates can be characterized by hesitant fuzzy soft set (F, A). The tabular representation of . hesitant fuzzy soft set (F, A) is shown in Table 1. In Table 1, we can see that the evaluation for an alternative to satisfy a criterion is represented by hesitant fuzzy set representing the grades given by the three interviewers.

Hesitant fuzzy soft set in decision making problems

	u_1	u_2	u_3	u_4
e_1	{0.6,0.70.,0.8}	{0.3,0.40.45}	{0.5, 0.6,0.7}	{0.7,0.8,0.8}
e_2	{0.7,0.8,0.85}	{0.6,0.7,0.65}	{0.7,0.8,0. 65}	{0.6,0.720,0.8}
e_3	{0.76,0.82,0.65}	{0.76,0.7,0.8}	{0.81,0.66,0.9}	{0.9,0.8,0.9}
e_4	{0.8,0.82,0.88}	{0.74,0.68,0.52}	{0.56,0.7,0.68}	{0.76,0.7,0.8}

Table 1

Hesitant fuzzy soft set in decision making problems

Then the score matrix (F_S, A) corresponds to (F, A) given in the table 1 is as follows:

	u_1	u_2	u_3	u_4
e_1	0.7	0.3833	0.6	0.766667
e_2	0.7833	0.65	0.7166	0.70667
e_3	0.7433	0.7533	0.79	0.866667
e_4	0.8333	0.6466	0.64667	0.7533

Table 2

The decision table for each person u_j obtained as follows

a_j	Values
a_1	.76497
a_2	.6083
a_3	.688
a_4	.7733

Table 3

Hesitant fuzzy soft set in decision making problems

Hesitant Intuitionistic Fuzzy Set (HIFS)

Definition (Beg and Rashid, 2014)

Given a set U. Suppose μ and μ' are functions applied to U return subsets of $[0,1]$ where $\mu(u)$ and $\mu'(u)$ are sets of some values in $[0,1]$.

An hesitant intuitionistic fuzzy set (HIFS) on U is a set

$$\tilde{\mathbf{X}} = \{< \mathbf{u}, \mu(\mathbf{u}), \mu'(\mathbf{u}) > |\mathbf{u} \in \mathbf{U}\}.$$

Values $\mu(u)$ and $\mu'(u)$ denote the possible memberhip degrees and non-membership degrees of the element $u \in U$ to the set $\tilde{X}$, which satisfy

$$max\{\mu(u)\} + min\{\mu'(u)\} \leq 1$$

and

$$min\{\mu(u)\} + max\{\mu'(u)\} \leq 1$$

For simplification,
$\tilde{\mu}(\mathbf{u}) = (\mu(\mathbf{u}), \mu'(\mathbf{u}))$ is called **an hesitant intuitionistic fuzzy element (HIFE)**, with

$$\mu(u) = \{\alpha_1, \ldots, \alpha_m | \alpha_i \in [0,1]\}$$

and

$$\mu'(u) = \{\alpha'_1, \ldots, \alpha'_n | \alpha'_i \in [0,1]\}.$$

The HIFS $\tilde{\mathbf{X}}$ can be written as

$$\tilde{\mathbf{X}} = \{< \mathbf{u}, \tilde{\mu}(\mathbf{u}) > | \mathbf{u} \in \mathbf{U}\}.$$

The set of all HIFSs on U is denoted by $\mathbf{I\tilde{H}}(\mathbf{U})$.

An HIFS $\tilde{X} = \{< u, \mu(u), \mu'(u) > | u \in U\}$ is called:

i) The **null HIFS**, denoted by $\tilde{\emptyset} = \{< u, \tilde{\mu}(u) > | u \in U\}$, if $\tilde{\mu}(u) = (\mu(u), \mu'(u)) = (\{0\}, \{1\})$.

ii) The **universal HIFS**, denoted by $\tilde{\mathbf{1}} = \{< u, \tilde{\mu}(u) > | u \in U\}$, if $\tilde{\mu}(u) = (\mu(u), \mu'(u)) = (\{1\}, \{0\})$.

Definition

Let $\tilde{\mu}_i(u) = (\mu_i(u), \mu_i'(u))$ be HIFEs with

$$\mu_1(u) = \{\alpha_1, \ldots, \alpha_m | \alpha_i \in [0,1]\},\ \mu_1'(u) = \{\alpha_1', \ldots, \alpha_n' | \alpha_i' \in [0,1]\}.$$

$$\mu_2(u) = \{\beta_1, \ldots, \beta_p | \beta_i \in [0,1]\},\ \mu_2'(u) = \{\beta_1', \ldots, \beta_q' | \beta_i' \in [0,1]\}.$$

We define the following HIFE operations:

i) $\tilde{\mu}^{\mathbf{c}}(\mathbf{u}) = (\mu'(u), \mu(u))$

ii) $\tilde{\mu}_{\mathbf{1}}(\mathbf{u}) \cup \tilde{\mu}_{\mathbf{2}}(\mathbf{u}) =$
$$\left(\cup_{\substack{\alpha_i \in \mu_1(u) \\ \beta_j \in \mu_2(u)}} \{max\{\alpha_i, \beta_j\}\}, \cup_{\substack{\alpha_i' \in \mu_1'(u) \\ \beta_j' \in \mu_2'(u)}} \{min\{\alpha_i', \beta_j'\}\} \right).$$

iii) $\tilde{\mu}_{\mathbf{1}}(\mathbf{u}) \cap \tilde{\mu}_{\mathbf{2}}(\mathbf{u}) =$
$$\left(\cup_{\substack{\alpha_i \in \mu_1(u) \\ \beta_j \in \mu_2(u)}} \{min\{\alpha_i, \beta_j\}\}, \cup_{\substack{\alpha_i' \in \mu_1'(u) \\ \beta_j' \in \mu_2'(u)}} \{max\{\alpha_i', \beta_j'\}\} \right).$$

The operations of HIFSs (Nazra et. al, 2018)

Definition

Given two HIFSs $\tilde{X} = \{< u, \tilde{\mu}(u) > | u \in U\}$,
$\tilde{Y} = \{< u, \tilde{\nu}(u) > | u \in U\}$ on U.
We define the following operations

i) $\tilde{\mathbf{X}}^{\mathbf{c}} = \{< u, \tilde{\mu}^c(u) > | u \in U\}$.

ii) $\tilde{\mathbf{X}} \cup \tilde{\mathbf{Y}} = \{< u, \tilde{\mu}(u) \cup \tilde{\nu}(u) > | u \in U\}$.

iii) $\tilde{\mathbf{X}} \cap \tilde{\mathbf{Y}} = \{< u, \tilde{\mu}(u) \cap \tilde{\nu}(u) > | u \in U\}$.

Proposition

Let $\tilde{\mu}_i$ be HIFEs. Then the following hold.

i) $(\tilde{\mu}_i^c(u))^c = \mu_i(u)$.

ii) $\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u) = \tilde{\mu}_2(u) \cup \tilde{\mu}_1(u)$.

iii) $\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u) = \tilde{\mu}_2(u) \cap \tilde{\mu}_1(u)$.

iv) $(\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u))^c = \tilde{\mu}_1^c(u) \cap \tilde{\mu}_2^c(u)$.

v) $(\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u))^c = \tilde{\mu}_1^c(u) \cup \tilde{\mu}_2^c(u)$.

vi) $\tilde{\mu}_1(u) \cap (\tilde{\mu}_2(u) \cap \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u)) \cap \tilde{\mu}_3(u)$.

vii) $\tilde{\mu}_1(u) \cup (\tilde{\mu}_2(u) \cup \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u)) \cup \tilde{\mu}_3(u)$.

viii) $\tilde{\mu}_1(u) \cup (\tilde{\mu}_2(u) \cap \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cup \tilde{\mu}_2(u)) \cap (\tilde{\mu}_1(u) \cup \tilde{\mu}_3(u))$.

ix) $\tilde{\mu}_1(u) \cap (\tilde{\mu}_2(u) \cup \tilde{\mu}_3(u)) = (\tilde{\mu}_1(u) \cap \tilde{\mu}_2(u)) \cup (\tilde{\mu}_1(u) \cap \tilde{\mu}_3(u))$.

Proposition

Let $\tilde{X}, \tilde{Y}, \tilde{Z}$ be HIFSs on U.

i) $(\tilde{X}^c)^c = \tilde{X}$.

ii) $\tilde{X} \cup \tilde{Y} = \tilde{Y} \cup \tilde{X}$.

iii) $\tilde{X} \cap \tilde{Y} = \tilde{Y} \cap \tilde{X}$.

iv) $(\tilde{X} \cup \tilde{Y})^c = \tilde{X}^c \cap \tilde{Y}^c$.

v) $(\tilde{X} \cap \tilde{Y})^c = \tilde{X}^c \cup \tilde{Y}^c$.

vi) $\tilde{X} \cup (\tilde{Y} \cap \tilde{Z}) = (\tilde{X} \cup \tilde{Y}) \cap (\tilde{X} \cup \tilde{Z})$.

vii) $\tilde{X} \cap (\tilde{Y} \cup \tilde{Z}) = (\tilde{X} \cap \tilde{Y}) \cup (\tilde{X} \cap \tilde{Z})$.

Generalized Hesitant Intuitionistic Fuzzy Soft Set (GHIFSS) (Nazra et. al, 2018)

Definition

Given $\widetilde{IH}(U)$ the collection of all HIFSs on U,
$\tilde{F}\colon E \to \widetilde{IH}(U)$ and α fuzzy set on E.
A pair $(\tilde{\mathbf{F}}_\alpha, \mathbf{E})$ is called a **GHIFSS over U**, where
$\tilde{F}_\alpha\colon E \to \widetilde{IH}(U) \times [0,1]$,
$\tilde{F}_\alpha(e) = (\tilde{F}(e), \alpha(e)) = (\{(u, \mu_e(u), \mu'_e(u)) | u \in U\}, \alpha(e))$
$\in \widetilde{IH}(U) \times [0,1]$ and
$(\tilde{\mathbf{F}}_\alpha, \mathbf{E}) = \{< e, \tilde{F}(e), \alpha(e) >\}$.

Definition

The **complement** of an GHIFSS $(\tilde{\mathbf{F}}_\alpha, \mathbf{E})$ is defined as
$(\tilde{\mathbf{F}}_\alpha, \mathbf{E})^{\mathbf{c}} = \{< \mathbf{e}, \tilde{\mathbf{F}}^{\mathbf{c}}(\mathbf{e}), \alpha^{\mathbf{c}}(\mathbf{e}) >\}$.

The operations of GHIFSSs (Nazra et. al, 2018)

Definition

Given two GHIFSSs $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, B)$ over U. We define two operations on such GHIFSSs as follows:

(1) $(\tilde{F}_\alpha, A) \wedge (\tilde{G}_\beta, B) := (\tilde{T}_\gamma, A \times B)$
$\tilde{T}_\gamma \colon A \times B \to \widetilde{IH}(U) \times [0,1]$ and
$\gamma \colon A \times B \to [0,1]$ which are defined by
$\tilde{T}(a,b) := \tilde{F}(a) \cap \tilde{G}(b)$,
$\gamma(a,b) = \alpha(a) \diamond \beta(b)$.

(2) $(\tilde{F}_\alpha, A) \vee (\tilde{G}_\beta, B) := (\tilde{N}_\psi, A \times B)$,
$\tilde{N}_\psi \colon A \times B \to \widetilde{IH}(U) \times [0,1]$ and
$\psi \colon A \times B \to [0,1]$ which are defined by
$\tilde{N}(a,b) := \tilde{F}(a) \cup \tilde{G}(b)$,
$\psi(a,b) = \alpha(a) \star \beta(b)$.

The properties of GHIFSSs (Nazra et. al, 2018)

Theorem

Given two GHIFSSs $(\tilde{F}_\alpha, A)$ *and* $(\tilde{G}_\beta, B)$ *over U. Then the following De Morgan's laws hold.*

i) $((\tilde{F}_\alpha, A) \wedge (\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \vee (\tilde{G}_\beta, B)^c$

ii) $((\tilde{F}_\alpha, A) \vee (\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \wedge (\tilde{G}_\beta, B)^c$

Proof.

By using the previous definition on GHIFSSs, proposition on HIFSs and Fuzzy sets;

i) $((\tilde{F}_\alpha, A) \wedge (\tilde{G}_\beta, B))^c = (\tilde{T}_\gamma, A \times B)^c$.

$\tilde{T}^c(a, b) = (\tilde{F}(a) \cap \tilde{G}(b))^c = \tilde{F}(a)^c \cup \tilde{G}^c(b) = \tilde{N}(a, b)$, and

$\gamma^c(a, b) = (\alpha(a) \diamond \beta(b))^c = \alpha^c(a) \star \beta^c(b) = \psi(a, b)$.

where $(\tilde{N}_\psi, A \times B) = \{(e, \tilde{N}(e), \psi(e))\} = (\tilde{F}_\alpha, A)^c \vee (\tilde{G}_\beta, B)^c$. $\square$

The properties of GHIFSSs-cont (Nazra et. al, 2018)

Theorem

Given three GHIFSSs $(\tilde{F}_\alpha, A)$, $(\tilde{K}_\beta, B)$ *and* $(\tilde{G}_\gamma, C)$ *over* U. *Then the following assosiative law holds:*

i) $(\tilde{F}_\alpha, A) \wedge ((\tilde{K}_\beta, B) \wedge (\tilde{G}_\gamma, C)) = ((\tilde{F}_\alpha, A) \wedge (\tilde{K}_\beta, B)) \wedge (\tilde{G}_\gamma, C)$

ii) $(\tilde{F}_\alpha, A) \vee ((\tilde{K}_\beta, B) \vee (\tilde{G}_\gamma, C)) = ((\tilde{F}_\alpha, A) \vee (\tilde{K}_\beta, B)) \vee (\tilde{G}_\gamma, C)$

Proof.

The proof of this theorem follows from the previous definition on GHIFSSs, proposition on HIFSs and Fuzzy sets. □

Note that the distributive law of GHIFSS does not hold because $A \times (B \times C) \neq (A \times B) \times (A \times C)$.

$$(\tilde{F}_{\alpha}, A) \wedge ((\tilde{K}_{\beta}, B) \vee (\tilde{G}_{\gamma}, C)) \neq$$
$$((\tilde{F}_{\alpha}, A) \wedge (\tilde{K}_{\beta}, B)) \vee ((\tilde{F}_{\alpha}, A) \wedge (\tilde{G}_{\gamma}, C)),$$

$$(\tilde{F}_{\alpha}, A) \vee ((\tilde{K}_{\beta}, B) \wedge (\tilde{G}_{\gamma}, C)) \neq$$
$$((\tilde{F}_{\alpha}, A) \vee (\tilde{K}_{\beta}, B)) \wedge ((\tilde{F}_{\alpha}, A) \vee (\tilde{G}_{\gamma}, C)).$$

Definition

Given two GHIFSSs $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, B)$ over U.
The **Union** of $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, B)$, denoted by $(\tilde{\mathbf{F}}_\alpha, \mathbf{A})\hat{\cup}(\tilde{\mathbf{G}}_\beta, \mathbf{B}) =: (\tilde{\mathbf{J}}_\gamma, \mathbf{C})$ is an GHIFSS, where $C = A \cup B$, and $\forall\ c \in C$,

$$\begin{aligned}
\tilde{J}(c) &= \tilde{F}(c), \quad \mathit{if} \;\; c \in A - B, \\
&= \tilde{G}(c), \quad \mathit{if} \;\; c \in B - A \\
&= \tilde{F}(c) \cup \tilde{G}(c), \;\; \mathit{if} \;\; c \in A \cap B. \\
\gamma(c) &= \alpha(c), \quad \mathit{if} \;\; c \in A - B, \\
&= \beta(c), \quad \mathit{if} \;\; c \in B - A \\
&= \alpha(c) \star \beta(c), \;\; \mathit{if} \;\; c \in A \cap B.
\end{aligned}$$

Definition

The **Intersection** of $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, B)$, denoted by $(\tilde{\mathbf{F}}_\alpha, \mathbf{A})\hat{\cap}(\tilde{\mathbf{G}}_\beta, \mathbf{B}) =: (\tilde{\mathbf{J}}_\gamma, \mathbf{C})$ is an GHIFSS,
where $C = A \cap B \neq \emptyset$, and $\forall\ c \in C$,
$\tilde{J}(c) = \tilde{F}(c) \cap \tilde{G}(c)$ and $\gamma(c) = \alpha(c) \diamond \beta(c)$.

Definition

A GHIFSS $(\tilde{F}_\alpha, A) = \{< e, \tilde{F}(e), \alpha(e) >\}$ is called:

i) The **null GHIFSS**, denoted by $(\tilde{\emptyset}_\emptyset, A)$, if $\tilde{F}(e) := \tilde{\emptyset}$, $\alpha(e) := 0$.
ii) The **universal GHIFSS**, denoted by $(\tilde{\mathbf{1}}_1, A)$, if $\tilde{F}(e) := \tilde{1}$, $\alpha(e) := 1$.

Note that if $A \neq B$ then the De Morgan's law is not satisfied for the union and the intersection of GHIFSSs.

Theorem

Given two GHIFSSs over U, $(\tilde{F}_\alpha, A)$ and $(\tilde{G}_\beta, A)$. Then

i) $((\tilde{F}, A)\hat{\cup}(\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \hat{\cap} (\tilde{G}_\beta, A)^c$

ii) $((\tilde{F}, A)\hat{\cap}(\tilde{G}_\beta, B))^c = (\tilde{F}_\alpha, A)^c \hat{\cup} (\tilde{G}_\beta, A)^c$.

Theorem

Given $\tilde{GH}$ the collection of all GHIFSSs over U, then the following holds

- *$\tilde{GH}$ is closed over operations $\hat{\cap}$ and $\hat{\cup}$*
- *$\tilde{GH}$ satisfies the associative law over operations $\hat{\cap}$ and $\hat{\cup}$*
- *there are GHIFSSs $(\tilde{\mathbf{1}}_{\mathbf{1}}, A)$ and $(\tilde{\emptyset}_\emptyset, A)$ such that $(\tilde{F}_\alpha, A)\hat{\cap}(\tilde{\mathbf{1}}_{\mathbf{1}}, A) = (\tilde{F}_\alpha, A)$ and $(\tilde{F}_\alpha, A)\hat{\cup}(\tilde{\emptyset}_\emptyset, A) = (\tilde{F}_\alpha, A)$ $\forall (\tilde{F}_\alpha, A) \in \tilde{GH}$.*

Fuzzy Sets and Open Problems

Fuzzy soft sets (FSS)
Intuitionistic FSS
Hesitant FSS
Interval-valued FSS
Interval-valued Intuitionistic FSS
Interval-valued Hesitant FSS
Interval-valued Intuitionistic Hesitant FSS
Entropy of FSS
Correlation of .....
Similarity of ....
Fuzzy N-Soft Sets (FNSS)
.......FNSS
..
Hesitant N-soft graphs,
Hesitant N-soft hypergraphs,
Hesitant Pythagorean fuzzy graphs

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Intuitionistic fuzzy recommender systems: An effective tool for medical diagnosis

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ABSTRACT

Medical diagnosis has been being considered as one of the important processes in clinical medicine that determines acquired diseases from some given symptoms. Enhancing the accuracy of diagnosis is the centralized focuses of researchers involving the uses of computerized techniques such as intuitionistic fuzzy sets (IFS) and recommender systems (RS). Based upon the observation that medical data are often imprecise, incomplete and vague so that using the standalone IFS and RS methods may not improve the accuracy of diagnosis, in this paper we consider the integration of IFS and RS into the proposed methodology and present a novel intuitionistic fuzzy recommender systems (IFRS) including: (i) new definitions of single-criterion and multi-criteria IFRS; (ii) new definitions of intuitionistic fuzzy matrix (IFM) and intuitionistic fuzzy composition matrix (IFCM); (iii) proposing intuitionistic fuzzy similarity matrix (IFSM), intuitionistic fuzzy similarity degree (IFSD) and the formulas to predict values on the basis of IFSD; (iv) a novel intuitionistic fuzzy collaborative filtering method so-called IFCF to predict the possible diseases. Experimental results reveal that IFCF obtains better accuracy than the standalone methods of IFS such as De et al., Szmidt and Kacprzyk, Samuel and Balamurugan and RS, e.g. Davis et al. and Hassan and Syed.

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A similarity measure of intuitionistic fuzzy soft sets and its application in medical diagnosis

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ABSTRACT

In this paper, a new similarity measure and a weighted similarity measure on intuitionistic fuzzy soft sets (IFSSs) are proposed and some of their basic properties are discussed. Using the proposed similarity measure, a relation ($\approx^{\alpha}$) between two IFSSs are defined and it is found that the defined relation is not an equivalence relation. Further, the effectiveness of the proposed similarity measure is demonstrated in a numerical example with the help of measure of performance and measure of error. Moreover, medical diagnosis problems have been exhibited through a hypothetical case study by using this proposed similarity measure. Finally, the proposed method is applied to 10 different medical data sets from UCI Machine Learning Repository datasets and its similarity measures are calculated. The corresponding performance measures, like, accuracy, sensitivity, specificity, ROC curves, AUC values, and *F*-measures are obtained and it is compared with the existing methods. This shows that the proposed method exhibits more accuracy, sensitivity and enhanced *F*-measures than the existing methods.

THANK YOU